McGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATH 264

ADVANCED CALCULUS

Examiner: Professor J.J Xu  Date: Monday December 17, 2007
Associate Examiner: Jeremy Van-Horn Morris       Time: 2:00 PM- 5:00 PM

INSTRUCTIONS

1. Please answer questions in the exam booklets provide.

2. This is a closed book examination. No books, crib sheets or lecture notes permitted.

3. Calculators are not permitted.

4. Use of a translation dictionary is permitted. No other types of dictionaries are permitted.

This exam comprises the cover page, two page of eight questions.
Final Examination of Math-264 Advanced Calculus  
(December 2007)

(1) Evaluate the double integral 
\[ \iint_D xy \, dA \]
where \( D \) is the triangular region with vertices (0, 0), (2, 0) and (0, 6).

(2) Find the volume of the region between the two paraboloids:
\[ (S_1) : \quad z = 10 - x^2 - y^2; \]
\[ (S_2) : \quad z = 2(x^2 + y^2 - 1). \]

(3) Find the work done by the force field
\[ \mathbf{F} = (y^2 \cos x + z^3)i + (2y \sin x - 4)j + (3xz^2 + 2)k \]
in moving a particle along the curve
\[ \{C_1\} : \begin{cases} 
  x = \sin^{-1} t \\
  y = 1 - 2t \\
  z = 3t - 1 
\end{cases} \quad (0 \leq t \leq 1), \]
from point (0, 1, -1) to the point \( \left( \frac{\pi}{2}, -1, 2 \right) \) and then returning along the straight line \( C_2 \) from \( \left( \frac{\pi}{2}, -1, 2 \right) \) to (0, 1, -1).

(4) Compute the flux of the vector field \( \mathbf{F} = xi + yj + zk \) upward through the part of the surface \( z = 5 - x^2 - y^2 \) lying above the plane \( z = 1 \).

(5) Let \( (D) \) be the region \( x^2 + y^2 + z^2 \leq 4a^2, \quad x^2 + y^2 \geq a^2 \). The surface \( (S) \) consists of a cylindrical part \( (S_1) \) and a spherical part \( (S_2) \). Evaluate the flux of 
\[ \mathbf{F} = (x + yz)i + (y - xz)j + (z - e^x \sin y)k \]
out of \( (D) \) through
(a) the whole surface \( (S) \),
(b) the surface \( (S_1) \),
(c) the surface \( (S_2) \).
(6) Evaluate

\[ \int \int_{(S)} \text{curl} \mathbf{F} \cdot \hat{N} \, dS \]

where \((S)\) is the surface \(x^2 + y^2 + 2(z-1)^2 = 6, z \geq 0, \hat{N}\) is the unit outward (away from the origin) normal on \((S)\) and

\[ \mathbf{F} = (xz - y^3 \cos z)i + x^3 e^z j + xyze^{x^2+y^2+z^2}k. \]

(7) Solve the following heat conduction equation by the method of separation of variables:

\[ \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad (0 < x < \pi, t > 0) \]

\[ u(0,t) = u(\pi,t) = 0, \quad (t > 0) \]

\[ u(x,0) = f(x), \quad (0 \leq x \leq \pi) \]

assuming that

(a) \( f(x) = 2 \sin 3x - \sin 5x; \)

(b) \( f(x) = \begin{cases} 
 x & 0 \leq x \leq \frac{\pi}{2} \\
 \pi - x & \frac{\pi}{2} \leq x \leq \pi.
\end{cases} \)

(8) Use Fourier series to solve the following wave equation:

\[ \frac{\partial^2 u}{\partial t^2} - 4 \frac{\partial^2 u}{\partial x^2} = 0, \quad (0 < x < 1, t > 0) \]

\[ u_x(0,t) = u_x(1,t) = 0, \quad (t > 0) \]

\[ u(x,0) = \cos^2 \pi x, \quad u_t(x,0) = \sin^2 \pi x \cos \pi x. \]