STUDENT NAME: _________________________

STUDENT NUMBER: ____________

FACULTY OF SCIENCE
FINAL EXAMINATION
MATHEMATICS 189-222A
CALCULUS III

Examiner: W. Jonsson Date: ?????????, December ????, 2010
Associate Examiner: N. Sancho Time: 9:00 AM - 12:00 PM

Instructions

1. Total number of points: 100.

2. No books, calculators or notes are allowed for the exam. Do not rip pages from the examination book.

3. There are 4 versions of this examination. This version belongs to Group 1.

4. Answers to Part I are to be entered on the machine readable sheet with a soft lead pencil.

5. Answers to part II are to be written in the space provided on the examination paper.

6. Your answers may contain $\pi$ or other expressions that cannot be computed without a calculator, e.g. $\ln 2$, $300^{1/2} + 13 \cdot 150^{-3/2}$.

7. All material (question papers, machine readable sheets) must be turned in.

8. Name, Student number and group number of your examination MUST be entered on the question paper and on the machine readable sheet.

GOOD LUCK!

Score Table

<table>
<thead>
<tr>
<th>Part I Multiple Choice</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Part II Problems</td>
<td>Points</td>
</tr>
<tr>
<td>1.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td></td>
</tr>
<tr>
<td>Total:</td>
<td></td>
</tr>
</tbody>
</table>

This exam comprises 7 pages, including this cover.
PART I. Multiple choice questions.

Each question is worth 3 points.

1. The equation of the plane tangent to the surface \( z = 2x^2 + y^2 \) at the point where \( x = 3, y = 2 \) is

   (a) \( z = -12x - 4y + 66 \),   (b) \( z = 12x - 4y - 6 \),   (c) \( z = -12x + 4y + 50 \),   (d) \( z = -12x + 4y - 30 \),   (e) \( z = 12x + 4y - 22 \).

2. The fourth degree Taylor polynomial of \( f(x) = x^2e^x \) centered at \( a = 0 \) is

   (a) \( 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} \),   (b) \( 1 + x + \frac{x^2}{2} + \frac{x^3}{6} \),   (c) \( x + x^2 + \frac{x^3}{2} + \frac{x^4}{6} \),   (d) \( x^2 + x^3 + \frac{x^4}{2} \),   (e) \( \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} \).

3. The vector \( \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \) is perpendicular to the vector \( 9\mathbf{i} - 3\mathbf{j} + c \cdot \mathbf{k} \) when

   (a) \( c = -1 \),   (b) \( c = 1 \),   (c) \( c = 2 \),   (d) \( c = 3 \),   (e) \( c = 0 \).

4. For any two vectors \( \mathbf{a}, \mathbf{b} \) the cross product \( (3\mathbf{a} + 2\mathbf{b}) \times \mathbf{a} \) is the same vector as

   (a) \( 2\mathbf{a} \times \mathbf{b} \),   (b) \( 2\mathbf{b} \times \mathbf{a} \),   (c) \( 3\mathbf{a} \times \mathbf{b} \),   (d) \( 3\mathbf{b} \times \mathbf{a} \),   (e) \( 5\mathbf{b} \times \mathbf{a} \).

5. The series \( \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n \)

   (a) has sum 1/2,   (b) has sum 1/3,   (c) has sum 2/3,   (d) diverges to \( \infty \),   (e) diverges to \( -\infty \).

6. The \( p \)-series \( \sum_{n=1}^{\infty} \frac{1}{n^p} \)

   (a) converges for \( p \geq 1 \) and diverges for \( p < 1 \),
   (b) converges for \( p > 1 \), diverges for \( p < 1 \) and we cannot say whether it converges or diverges for \( p = 1 \),
   (c) diverges for \( p \geq 1 \) and converges for \( p < 1 \),
   (d) diverges for \( p > 1 \) and converges for \( p \leq 1 \),
   (e) converges for \( p > 1 \) and diverges for \( p \leq 1 \).

7. The power series \( \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \) represents the function

   (a) \( e^x \),   (b) \( \sin x \),   (c) \( \cos x \),   (d) \( \arctan x \),   (e) \( \tan x \).
8. The power series \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)!(n+1)!2^{2n+1}} \)

(a) converges only for \( x = 0 \), (b) has radius of convergence 2,
(c) converges for all real numbers \( x \), (d) has interval of convergence \(-4 < x < 4\),
(e) has radius of convergence 1.

9. A particle is moving along the trajectory \( \mathbf{r}(t) = 2 \cos t \mathbf{i} + 3 \sin t \mathbf{j} + t \mathbf{k} \). At time \( t = \pi/2 \) the velocity vector \( \mathbf{v}(\pi/2) \) and the acceleration vector \( \mathbf{a}(\pi/2) \) are

(a) \( \mathbf{v}(\pi/2) = -2\mathbf{i} + \mathbf{k} \), and \( \mathbf{a}(\pi/2) = -3\mathbf{j} \),
(b) \( \mathbf{v}(\pi/2) = -2\mathbf{j} + \mathbf{k} \), and \( \mathbf{a}(\pi/2) = -3\mathbf{i} \),
(c) \( \mathbf{v}(\pi/2) = 2\mathbf{i} + \mathbf{k} \), and \( \mathbf{a}(\pi/2) = 3\mathbf{j} \),
(d) \( \mathbf{v}(\pi/2) = 2\mathbf{j} + \mathbf{k} \), and \( \mathbf{a}(\pi/2) = 3\mathbf{i} \),
(e) \( \mathbf{v}(\pi/2) = 3\mathbf{i} + \mathbf{k} \), and \( \mathbf{a}(\pi/2) = 2\mathbf{j} \).

10. The directional derivative of the function \( f(x, y) = x^2 + 2y^2 \) in the direction of the unit vector \( \mathbf{u} = (1 - j)/\sqrt{2} \) and at the point \( (2, 3) \) is

(a) 0, (b) \( 2/\sqrt{2} \), (c) \(-4/\sqrt{2}\), (d) \( 6/\sqrt{2} \), (e) \(-8/\sqrt{2}\).

11. The series expansion of \( \int e^{x^2} \, dx \) is

(a) \( 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots \),
(b) \( x + x^3 + \frac{x^5}{2!} + \frac{x^7}{3!} + \frac{x^9}{4!} + \cdots \),
(c) \( x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{3!7} + \frac{x^9}{4!9} + \cdots \),
(d) \( x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \frac{x^9}{9} + \cdots \),
(e) \( x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5} + \cdots \).

12. At the point \( (2, 1) \) the direction in which the function \( f(x, y) = \frac{x^2}{4} + y^2 \) has the maximum rate of change is given by the vector

(a) \(-\mathbf{i} - 2\mathbf{j}\), (b) \(-\mathbf{i} + 2\mathbf{j}\), (c) \(2\mathbf{i} + \mathbf{j}\), (d) \( \mathbf{i} + 2\mathbf{j}\), (e) \( \mathbf{i} - 2\mathbf{j}\).
13. The equation of the plane tangent to the surface $e^x + 2y + \sin z = 0$ at the point $(0, -1/2, 0)$ is
   (a) $x + 2y + z = -1$,  
   (b) $x - 2y + z = 1$,  
   (c) $2x + y + z = -1/2$,  
   (d) $2x - y + z = -1/2$,  
   (e) $x + 2y - z = -1$.

14. The tangent plane to the level surface $x^2 + \frac{y^2}{4} + z^2 = 3$ of the function $F(x, y, z) = x^2 + \frac{y^2}{4} + z^2$ at the point $(1, 2, 1)$ has equation
   (a) $-2x + y + 2z = 2$,  
   (b) $2x - y + 2z = 2$,  
   (c) $2x - y - 2z = -2$,  
   (d) $2x + y + 2z = 6$,  
   (e) $2x + y - 2z = 2$.

END OF PART I
PART II.

1. (15 points) Let $f(x,y)$ be a differentiable function and $x = 2s \cos t$, $y = 3s \sin t$.

(a) Find expressions for $\frac{\partial f}{\partial s}$, $\frac{\partial f}{\partial t}$ in terms of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(b) Show that, for the same change of variables, we have

$$4 \left( \frac{\partial f}{\partial x} \right)^2 + 9 \left( \frac{\partial f}{\partial y} \right)^2 = \left( \frac{\partial f}{\partial s} \right)^2 + \frac{1}{s^2} \left( \frac{\partial f}{\partial t} \right)^2.$$
2. (16 points) Consider the function \( f(x, y) = 2x^2 + 8xy + y^4 \).

(a) Find the critical points and classify them as local maxima, local minima and saddle points.

(b) For the same function approximate \( f(1.99, 1.02) \) with the help of differentials.
3. (9 points) Change the order of integration and evaluate the following integral
\[ \int_{0}^{1} \int_{x^2}^{1} x^3 \sin(y^3) \, dy \, dx. \]

4. (8 points) Find the volume under the cone \( z = \sqrt{x^2 + y^2} \) and above the region in the \( xy \)-plane lying between the two circles \( x^2 + y^2 = 1 \), \( x^2 + y^2 = 4 \). Use polar coordinates.
5. (10 points) Find the extreme values of the function

\[ f(x, y) = 2x + y \]

on the ellipse

\[ \frac{x^2}{2} + \frac{y^2}{8} = 1. \]

Which are maxima and which are minima?
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Continuation of solution for problem: ____.
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