Final Examination

McGill University
Faculty of Science

Mathematics 141 FALL 09

Calculus 2
Date: Friday, Dec. 11th 2009
Time: 14:00 – 17:00 hours

EXAMINER: Prof. N. Sancho
ASSOCIATE EXAMINER: Prof. W. Brown

FAMILY NAME:
OTHER NAME:
STUDENT NUMBER:

INSTRUCTIONS

1. Calculators are not permitted
2. You can continue your work in the preceding (or facing) page or in the pages at the end of the booklet.
3. This examination booklet consists of a cover page, plus 14 pages; pages 11, 12, 13 and 14 are blank. You must not tear pages from this booklet.
4. Dictionaries are not allowed
5. This is a closed book examination.

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1. SHOW ALL YOUR WORK

(a) [2 marks] If \( F(x) = \int_{0}^{\sqrt{x}} \frac{t^2}{1+t^4} \, dt \) find the derivative of \( F(x) \).

(b) [3 marks] Let \( F(x) = 2x + \int_{x}^{0} \frac{\sin 2t}{1+t^4} \, dt \). Determine \( F(0); F'(0); F''(0) \).

(c) [5 marks] Sketch the region bounded by the curves \( y = 12 - x^2 \) and \( y = x^2 - 6 \) and find its area.
2. SHOW ALL YOUR WORK
[7 marks] Sketch the region bounded by the curves; $x = y^3$, $x = 8$, $y = 0$ and find the volume of the solid generated by revolving the region about the y-axis.
3. SHOW ALL YOUR WORK

(a) [8 marks] Calculate \( \int \frac{x + 4}{x^2 + 2x + 5} \, dx \).

(b) [7 marks] Calculate \( \int (\ln x)^2 \, dx \).
4. SHOW ALL YOUR WORK
   (a) [8 marks] Prove the reduction formula for all integers $n \geq 2$
   $\int \sec^n x \, dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx.$
   (b) [7 marks] Evaluate $\int \sec^5 x \tan^2 x \, dx.$
5. SHOW ALL YOUR WORK
[10 marks] Find the exact area of the surface obtained by rotating the curve about the x-axis: \( x = t^2, y = t^2, 0 \leq t \leq 1 \).
6. SHOW ALL YOUR WORK
   
   (a) [10 marks] Evaluate \( \int \frac{\sin^3(\ln x) \cos^3(\ln x)}{x} \, dx \)
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6. SHOW ALL YOUR WORK
   (b) [5 marks] Evaluate the improper integral \( \int_{\frac{3}{2}}^{3} \frac{dx}{\sqrt{3-x}} \).
7. SHOW ALL YOUR WORK

(a) [5 MARKS] find the arc length of the curve from $x = 1$ to $x = 2$ of $y = x^2 - \frac{1}{8} \ln x$. 
7. SHOW ALL YOUR WORK
   (b) [10 MARKS] Find the area of the inner loop of $r = 1 + 2\sin \theta$. (Hint: sketch the curve)
8. SHOW ALL YOUR WORK

(a) [4 marks] Determine whether the given series converges or diverges \( \sum_{n=1}^{\infty} \frac{n!}{n^2 e^n} \).

(b) [3 marks] Determine whether the geometric series \( \sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n} \) is convergent or divergent. If it is convergent find its sum.

(c) [6 marks] Determine whether the series converges absolutely, converges conditionally or diverges: \( \sum_{n=0}^{\infty} \frac{\cos(n\pi)}{(n+1)\ln(n+1)} \).
You must refer to this continuation page on this page where the problem is printed.