

1. Using Gaussian elimination, bring the matrix A to reduced row echelon form while keeping track of a book-keeping matrix:

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \end{pmatrix}$$

- (a) Find all the dependence relations on the rows of A .
 - (b) Find a basis for the row space of A .
 - (c) Find a basis for the column space from among the columns of A .
 - (d) Find a basis for the column null space.
 - (e) Is the vector $(1, -3, 0)^t$ in the column space of A ? If it is, display it as a linear combination of the basis found in (c). If not, why not?
 - (f) Is the vector $(1, -1, 2, 3)$ in the row space? If it is, display it as a linear combination of the basis found in (b). If not, why not?
2. Prove the following theorems.
- (a) State the axioms for the determinant function and, using them, prove that if two rows are equal, then the value of the determinant is zero.
 - (b) Give the definition of a Hermitian matrix A . Show that the eigenvalues of a Hermitian matrix A are real.
3. Consider the system of linear differential equations $\mathbf{v}' = A\mathbf{v}$ where

$$A = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$$

Notice that 3 is an eigenvalue of A .

- (a) Find the eigenvalues and eigenvectors of A .
- (b) Find a fundamental matrix for the differential equation.
- (c) Using this fundamental matrix, calculate the exponential e^{At}
- (d) Solve the non-homogeneous system $\mathbf{w}' = A\mathbf{w} + e^{-t} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ with the initial condition $\mathbf{w}(0) = \mathbf{0}$. The answer may be left in the form of an integral.

4. Consider the quadratic form

$$2x^2 - 4xy + 5y^2$$

- (a) Using a change of basis (display the transition matrix P), diagonalise and classify this quadratic form.
- (b) Hence or otherwise, evaluate the double integral

$$\int_0^\infty \int_0^\infty e^{-(2x^2 - 4xy + 5y^2)} dx dy.$$

McGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATHEMATICS 189-270A

APPLIED LINEAR ALGEBRA

Examiner: Professor W. Jonsson

Date: Monday, December 7, 1998

Associate Examiner: Professor S.A. Maslowe

Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

FACULTY STANDARD CALCULATORS PERMITTED

This exam comprises this cover and 1 page of questions.