

- Using Gaussian elimination, bring the matrix  $A$  to reduced row echelon form while keeping track of a book-keeping matrix:

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \end{pmatrix}$$

- Find all the dependence relations on the rows of  $A$ .
  - Find a basis for the row space of  $A$ .
  - Find a basis for the column space from among the columns of  $A$ .
  - Find a basis for the column null space.
  - Is the vector  $(1, -3, 0)^t$  in the column space of  $A$ ? If it is, display it as a linear combination of the basis found in (c). If not, why not?
  - Is the vector  $(1, -1, 2, 3)$  in the row space? If it is, display it as a linear combination of the basis found in (b). If not, why not?
- Prove the following theorems.
    - State the axioms for the determinant function and, using them, prove that if two rows are equal, then the value of the determinant is zero.
    - Give the definition of a Hermitian matrix  $A$ . Show that the eigenvalues of a Hermitian matrix  $A$  are real.
  - Consider the system of linear differential equations  $\mathbf{v}' = A\mathbf{v}$  where

$$A = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$$

Notice that 3 is an eigenvalue of  $A$ .

- Find the eigenvalues and eigenvectors of  $A$ .
  - Find a fundamental matrix for the differential equation.
  - Using this fundamental matrix, calculate the exponential  $e^{At}$
  - Solve the non-homogeneous system  $\mathbf{w}' = A\mathbf{w} + e^{-t} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$  with the initial condition  $\mathbf{w}(0) = \mathbf{0}$ . The answer may be left in the form of an integral.
- Consider the quadratic form

$$2x^2 - 4xy + 5y^2$$

- Using a change of basis (display the transition matrix  $P$ ), diagonalise and classify this quadratic form.
- Hence or otherwise, evaluate the double integral

$$\int_0^\infty \int_0^\infty e^{-(2x^2 - 4xy + 5y^2)} dx dy.$$

McGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATHEMATICS 189-270A

APPLIED LINEAR ALGEBRA

Examiner: Professor W. Jonsson  
Associate Examiner: Professor S.A. Maslowe

Date: Monday, December 7, 1998  
Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

FACULTY STANDARD CALCULATORS PERMITTED

This exam comprises this cover and 1 page of questions.